M. GOLOVANOVA, V. LEBEDCHENKO

USING THE ECONOMIC AND MATHEMATICAL MODELS FOR DETERMINING THE MARKET CAPACITY

The subject matter of the article is economic and mathematical forecasting models of the market capacity based on the system of influential factors. The goal is to systematize and substantiate theoretical and methodological approaches to forecasting the market capacity in the conditions of the current unstable state of the economy. The following tasks are solved: the factors that influence on the development and amount of the market capacity are completely systematized; peculiarities, advantages and disadvantages of applying available economic and mathematical forecasting models of the market capacity are considered; the necessity for taking into account the structural changes in a number of consumers by age groups is emphasized; the features of the influence of the consumption of luxury goods and goods of prime necessity were is revealed taking into account the return on the change in the market capacity. The methods used are – the system analysis, the factor analysis, the regression and correlation analysis, the methods of cumulative curves. The following results are obtained – the principles of building economic and mathematical forecasting models of the market capacity are considered; the factors that influence on the market capacity at micro and macro level and have a random or systematic nature of the impact were systematized; advantages, disadvantages and peculiarities of multi-factor and single-factor models of the market capacity, in particular, cumulative curves are revealed; compulsory registration of demographic factors (changing the structure of age groups in time) in multi-factor models is suggested; the alternative use of cumulative curves of demand on the groups of luxury goods and goods of prime necessity depending on income is shown. Conclusions. Taking into consideration structural changes in a number of consumers in time, the features of consumption of luxury goods and goods of prime necessity using the factors of the elasticity of demand on income and other significant factors enable mathematical describing the consumption of the basic groups of goods and services, forecasting the amount of their market capacity, determining the critical values of the factors that can completely stop consuming products of a certain group or reduce the consumption to the minimum level.

Keywords: market capacity, economic and mathematical model, the elasticity of demand, income, consumption, price, demand.

Introduction

Dynamic changes that have taken place in the economy over the recent decades are so intensive and fast that the theoretical understanding of new market realities lies far behind their implementation. The active activity of enterprises or their survival under a turbulent economy requires using adequate and efficient mechanisms of the economic regulation of development. So, one of the most important and responsible issues of the activity of enterprises is the awareness of the market capacity, the amount of demand for the goods of an enterprise. All this becomes especially topical in the context of the crisis, the growth of costs, the reduction in purchasing power, the saturation of markets, increased supply of certain types of goods, the intensification of competition.

The awareness and forecast of market indicators and its capacity are very important, they are of great interest to the market participants as they allow companies to understand clearly, strengthen and expand their positions on the market.

It should be noted that today the need for information on the capacity, size and shares of the market has already been formed; there are many companies that carry out similar marketing research. At the same time, some enterprises estimate the capacity of the market on their own.

The analysis of literary sources and problem statement

The issues of the technique of the methodology of calculating the market capacity were studied in the works written by G. Azoyev and V. Khrutsky, P. Zavyalov, S. Lavrov, E. Golubkov, I. Lipsitz, S. Pivovarov, E.I. Lunin, J.-J. Lamben, V. Praude, F. Kotler, D. Evans, G. Chandezon, K. Schmittchoff, G. Harding and other authors. Such scientists as G. Pospelov, V. Leontiev, L. Abalkin, I. Bestuzhev-Lady, N. Malchotry, G. Tayla, H. Schumann, D. Hicks, C. Arrow contributed to developing the forecasting methodology.

However, the methodological issues of forecasting the market capacity are insufficiently worked out in the mentioned works, factors that determine the market capacity are insufficiently indicated, the influence of micro and macro factors on the development of market potential is not fully reflected, the specificity of the structural and regional features of the markets is not taken into account, the demographic factor of structural changes in a number of consumers by age groups is not fully elaborated, the impact of consumers’ incomes in terms of different conditions and development of the economy. Theoretical and practical problems of forecasting the market capacity need to be understood on the principles of market relations. It should be noted that today there are many economic and mathematical methods for calculating the market capacity, while there are no universal ones. The mentioned above determined the subject of this article.

The goal and objectives of the research

The goal of the article is to systematize and substantiate theoretical and methodological approaches to forecasting the market capacity in the conditions of the current unstable state of the economies of individual countries.

The objectives are:
- to systematize factors that affect the development and amount of the market capacity;
- to consider the peculiarities, advantages and
disadvantages of using available economic and mathematical models of forecasting the market capacity;
- to consider the necessity for taking into account structural changes in a number of consumers by age groups in the forecasting models;
- to identify the peculiarities of influence on changing the market capacity of the consumption of luxury goods and goods of primary necessity using the indicators of the elasticity of demand taking into account the stratifications of the society in different countries.

**The results of the research**

The choice of the method for determining the capacity of a product or company market depends on the specifics of the business, the urgency and importance of the task, the accessibility of information, labour and financial resources, patterns for the promotion of goods to a consumer; each group of goods needs an individual approach.

The analysis of the current practice of forecasting market processes enables highlighting a number of features of building the economic and mathematical models of the market capacity whose choice and use is conditioned not only by the objectives of the research but also by the quality of available empirical data [1, 2].

Various methods are used to forecast the market capacity [3-5]. It should be noted that the application of any particular method does not improve the accuracy of the forecast, does not provide an integrated approach and reduce the risk of making managerial decisions on the direction of the development of an enterprise.

Building economic and mathematical models for determining (forecasting) the capacity of the market and demand should be based on the following principles [6, 7]:

1) analyzing and modelling the structure of consumption per the totality of groups of goods and services presented in the market and necessary to meet various human needs; the structure of consumption should be considered in two aspects:
   - as the amount of consumption of a certain group of products in physical terms;
   - as the level of expenses for purchasing a certain group of products;

2) substantiating the lower and upper regulatory boundaries of consuming goods intended to meet the needs of people who are in the lower stages of the needs hierarchy. Due to these boundaries, the main object of the analysis of the market capacity is the consumption of a certain group of products in physical terms whose values exist and vary within a certain interval. At the same time, at a certain point of an interval between the lower and upper consumption standards the moment of market saturation takes place, it depends on the specific features of a particular group of goods or services. After saturation, the subsequent change in the independent factor causes a decrease in the rates of consumption growth of a certain group of goods;

3) taking into account the factors that determine the change in the market capacity and the amount of the demand for a particular group of goods;

4) ranking different groups of goods and services by the degree of their value for consumers, which enables determining the order of their purchasing when changing other considered factors of product consumption;

5) determining the weight coefficients of the value of goods (commodity groups) for consumers;

6) taking into account the size of the market – the data of state statistics should be used to assess the size of the market in large areas (country, region). On small territories (district, city) field research can be carried out as market statistics are not usually conducted;

7) substantiating the influence of different levels of average income per capita and average prices on the amount of the market capacity and demand.

Let summarize and categorize the well-known factors that need to be taken into account while determining the market capacity (some factors are often taken into account; others are not considered at all) [8]:

- the geographic boundaries of the market – a district of the city (Kharkiv, Kyivsky district, Saltivka area), a city (the city of Kharkiv), several cities Kharkiv, Kyiv, Odesa, one region or several regions of the country (Slobozhanschina, Zakarpattia), Ukraine, foreign markets (Poland market, European market), etc.;

- the time interval of marketability – a season, a quarter, a year, etc. A year is widely viewed as the interval as the majority of goods and services are seasonal within a period that is usually equal to a year;

- the homogeneity of the market – if the market is heterogeneous and is divided into segments (buying behaviour among the segments varies considerably), the market capacity needs to be determined for each segment separately. The resulting indicators should be later aggregated;

- the units of measurement – natural or value; when calculating the cost index of the market it is necessary to determine what prices (consumers or producers) and which currency to take into account. It is recommended that producer prices be used in the b2b segment, consumer prices be used in the b2c segment, the national currency – hryvnia – should be used on the domestic market and euros or dollars should be used on the international market;

- demographic characteristics of consumers – the quantity, density, social composition of the population and its composition by gender and age; general market conditions; a kind and type of goods, commodity groups, their aggregation; the amount and structure of the product offering; enterprises-representatives; the quality of products and changes in the quality of goods; the amount of imports of a particular product or group; the demand for goods and the elasticity of demand;

- the level of prices and changes in the average market price for the whole population or individual product groups; the ratio of prices for goods that are presented on the market and are necessary to meet different types of human needs;
- the achieved standard of living and needs of the population; changing the population lifestyle;
- the incomes of the population, the purchasing power of the population; a share of the income that a consumer is ready to spend on purchasing a particular product;
- the degree of value of a certain type of goods and services for consumers, which is conditioned by the need for a specific group of products to meet the needs of people who are at different hierarchical stages;
- the stage of the product life cycle and the corresponding type of a consumer (innovators, early majority, later majority, those who lag behind); the degree of development and saturation of a particular market, the probability of similar goods or other goods with similar properties (characteristics) on the market;
- changes in macroeconomic indicators – business activity; prices for energy; the leading indicator, that is, the change of the target market or another one that responds to the future change in the market capacity with a predetermined time lag;
- the advent, expansion and accessibility of new technologies;
- the intensity of marketing efforts of producers and sellers; the efficiency of promotion on the market; the cost-effectiveness of advertising; the state of the marketing, sales and service network;
- specific factors that determine the development of markets for individual products while each market may have characteristics that are specific only to it. A specific factor in the degree of impact may be crucial for generating and developing the demand and offer of a particular product. Specific factors include natural and climatic conditions, changes in fashion; national and everyday traditions (for the markets of fabrics, clothes and footwear); the achieved level of prosperity; the terms of physical and moral wear; the size and composition of a consumer's wardrobe; the increased housebuilding and so on.

These factors have a different impact because when the income changes by a certain value and prices change by the same value but with the opposite sign, the change in the volume of product consumption is not the same. In this case, consumers' incomes and the level of prices on the amount of the market capacity of a particular group of goods or services can have the overall impact while combining them into one indicator that is determined as the ratio of factors, which characterizes the purchasing power of population to buy goods from the target group of products.

Thus, a great number and variety of factors should be taken into account in economics and mathematical models and the effects of individual factors cannot be distinguished.

Random and systematic factors should be distinguished [6]. Random factors display the individual tastes of consumers and their environment, customs, the peculiarity of the circumstances which surround them, in other words, all things that are related to the personality and behaviour of a consumer. Systematic factors characterize the size of the family, gender and age of family members, profession and social group membership; the effect of these factors can be eliminated by classifying families and grouping them but anticipating all the factors in advance is impossible.

Factors can be distinguished experimentally, then consumer budgets can be clearly classified. Distinguishing families according to the different age, size, the level of income, etc. enables determining the impact of factors related to a small intra-group dispersion.

After eliminating all the distinguished systematic factors by grouping and selecting other factors, all the factors that have not been taken into account should be considered as random. This allow the manufacturing companies to quantify the impact of factor changes on the amount of the market capacity of the manufactured products, to anticipate the change in the scale and duration of the life cycle of the market and, consequently, to respond to changing market conditions in the most effective way taking into account the final results of the organization activities.

The dependencies of the market capacity on macroeconomic parameters can be modelled by building factor forecasting models that characterize the dependence of consumption on the level and composition of monetary incomes, the price level and the ratio of price indices as well as on the social and the demographic composition and size of households. The essence of this approach lies in the fact that the market capacity is presented as a function of one or several factors. To build this class of models, the income differentiation and the elasticity of consumption of goods by all the population should be studied, certain groups of people with different paying capacity should be analyzed.

**Multi-factor models of determining the market capacity.**

Over long time intervals, market phenomena and processes are determined by a combination of factors, their overall impact on the amount of the market capacity is taken into account by multi-factor models of forecasting

\[ S = \varphi(X) = \varphi(x_1, x_2, \ldots, x_n). \]

(1)

where \( x_1, x_2, \ldots, x_n \) are the factors that influence the market capacity.

If a model is built for a short period of time, individual factors can be considered as constants.

For forecasting of the market capacity, works [8, 9] suggest using such multi-factor models that take into account the available statistical data:

1) depending on the income obtained within the current and previous periods

\[ S_t = a_0 + a_1 I_t + a_2 I_{t-1}, \]

(2)

where \( S_t \) is the market capacity in the planned period; \( I_t \) is the level of customers' income in the planned period; \( I_{t-1} \) is the level of income in the period that preceding the planned period; \( a_0, a_1, a_2 \) are the coefficients of regression.
2) depending on the income of consumers obtained within the current period and the demand within the period preceding the planned period

\[ S_t = a_0 + a_1 I_t + a_2 S_{t-1}, \]

where \( S_{t-1} \) is the market capacity within the period preceding the planned one;

3) depending on the level of consumers’ income within the period preceding the planned one and on the maximum values of consumers’ demand within a certain previous time period

\[ S_t = a_0 + a_1 I_{t-1} + a_2 E_{\text{max}}, \]

where \( E_{\text{max}} \) is the maximum value of the market demand within a certain time period that precedes the planned period of time;

4) depending on the current income of consumers and on the average level of prices for all the consumer goods within the target period

\[ S_t = a_0 + a_1 I_t + a_2 P_t, \]

where \( P_t \) is the average level of prices for all the consumer goods within the target period.

Dynamic models for calculating the demand (the market capacity).

The selection of factors that determine the dynamics of the market capacity in the context of building macroeconomic models has certain features. Analytically, there is a system of factors that are causally related to the market capacity indicator. However, mathematically, not all of them can be adequately taken into account in the model since macro-factors have a complex hierarchical structure and can be measured using various indicators. For example, the purchasing power of people’s income depends on incomes and on inflation rate at the same time.

Despite the fact that the market capacity is changing dynamically and in different periods, the market situation can be different but the general trends that reflect the specifics of the type of consumption remain unchanged. At the same time, the change in the nature of processes that determine the behaviour of consumers in the dynamics affects their statistical certainty. Consequently, an index that did not have a significant decisive influence in the past is considered as a weak signal. In case, when it cannot be used in future, the available indices should be retrospectively reviewed as those that characterize differences in the typology of consumers’ behaviour.

1. Basing on the dynamic model of consumption with due regard for stocks (H. S. Houthakker and L. J. Taylor):

\[ C_t = a_0 + a_1 Z + a_2 I + \varepsilon, \]

where \( C_t \) is the consumption; \( Z \) is the stock of goods or the habit of their consumption habit; \( I \) is the income; \( \varepsilon \) is a random component.

2. Depending on the price and time. The simplest models of the demand depending on the time and price include the following one

\[ C_t = a_0 + a_1 P_t + a_2 t, \]

where \( P_t \) is the price; \( t \) is the factor of time.

3. Taking into account the ratio of price indices

\[ \ln C_j = \ln a_0 + a_1 \ln P_j + a_2 \ln J_p, \]

where \( C_j \) is the demand for a certain item of goods; \( P_j \) is the price for a certain item of goods; \( J_p \) is the comparative index of prices that characterizes the ratio of changes in prices for a certain item of goods and the general index of prices.

The dependence of the amount of consumption on the income is represented by the function of consumption on the macro level, and this fact should be taken into consideration. In his works, J. M. Keynes illustrates the ratio of the generalized indices of income, consumption, capital investments, savings – when the income increases, the consumption also increases but slower. At a certain level of consumption, people start to save.

Actual data on the market capacity show how consumers are supplied with different types of goods and services in each specific period of time. The structure of consumers’ demand in any period of time and its change under the influence of various factors is the reflection of objectively existing limits of the consumer behaviour in the area of distributing their income for purchasing one or another group of products. Within these boundaries of consumption, different types of human needs are met with the help of a certain group of products. The following should be taken into consideration:

- the lower boundary characterizes the amount of consumption of products necessary to meet minimum demand;
- the upper boundary is the natural boundary of product saturation is conditioned by the law of diminishing marginal utility; that means that consumers’ needs for certain groups of goods or services are entirely met or their needs cannot be met better by increasing the amounts of purchasing.

Depending on the amount of available statistical information, the patterns of the market capacity change \( S \) as the function of changing any factor from the mentioned ones and forecasting the market capacity can be analyzed using the coefficients of the demand elasticity:

\[ S = f\left( E_p, E_{AB}, E_I, E_K \right), \]

where \( E_p \) is the coefficient of price elasticity; \( E_{AB} \) is the coefficient of cross-price elasticity; \( E_I \) is the coefficient of the consumers’ income elasticity; \( E_K \) is the coefficient of the goods quality elasticity.

The goods are distinguished as those of

1) low elasticity (0 < \( E < 1 \)); this group includes the goods of primary necessity (bread, salt, matches); as income \( I \) grows, the increase in demand for essential goods \( Q \) gets slower and has the saturation boundary;
2) moderate elasticity (E = 1 or E ≈ 1); this group includes the goods of secondary necessity; the function has the saturation boundary but of a higher order; the demand for such goods arises after the income reaches a certain desired value;

3) elasticity (E > 1); this group includes luxury goods; the function has no boundaries, the demand for luxury goods arises after the income exceeds a certain level of income and the increase in demand outruns the increase in income.

A drawback of multi-factor models is the fact that the limited volume of statistical information about the amount of the market capacity and its determinants leads to significant errors in the calculations of the current value and forecasts. This is due to the fact that the value of the target variables for a basic period of time can turn out to be indistinctive, false for a particular market when empirical data is accumulated.

The above models also have significant limitations. Forecasting the demand at the following moment of time, using statistics of historical data, can lead to completely opposite results; this can happen when the demand is seasonal (taking into account the seasonal wave requires other approaches), when the goods moved from one phase of the life cycle to another, when the consumption of goods is very sensitive to inflationary processes and so on.

It should also be noted that the above models do not take into account the state of the demographic situation on the target market. The demographic structure of consumers is very variable in age, gender, the density of residence due to active migration and emigration processes, structural shifts in the age pyramid (fig. 1).

**Fig. 1. The structure of Ukrainian population according to the gender and age (according to the State Statistics Committee of Ukraine [10])**

The information on the gender and age distribution of the residents of a country, region, or any administrative-territorial entity is demanded practically in all spheres of public life that are related to human life and activities. In marketing, following F. Kotler, scientists consider the age of consumers to be very significant and refer it to a group of personal factors. So, A.S. Posipanova noted in her monograph [11] that social and demographic factors (gender, age, social status) have a significant influence on consumer behaviour (demand) and the influence of price, income and quantity of goods is insignificant since they are included in traditional economic models. The author also emphasizes that income affects the choice of goods but not on preferences. D. Borzekovski [12], V. Stefan [13] found a significant difference not only in the preferences of children and adults but also in the attributes of preferences, their range (children have significantly narrower one) and the rate of development (children develop preferences more quickly).

The influence of age on consumption patterns is manifested in the choice and frequency of consumption of goods and services. So, according to marketing research, in 2007 young people of 17-23 years often made use of beauty salon services (26.7%), entertainment centres (17.8%), cafes and bars (11.5%). This is a very large proportion of the population. In 2017 these consumers moved to another group (a group of 30-year-old people), and now their views and desires are aimed at the services of banks (45.8%), entertainment centres (28.7%). At the same time, in 2017 the category of 20-year-old consumers of services mentioned above decreased almost twice (fig. 1): in 2007 – 791 thousand people, in 2017 – 446 thousand people.

It should be noted that information on the number of children already born enables forecasting the consumption of a group of children’s products (baby carriages, baby beds, linens, diapers, baby food, etc.) in the short term and makes it possible to forecast the consumption of certain goods and services in the future.

**Single-factor models of calculating the market capacity taking income into account.**

Despite the fact that multi-factor models of the market capacity forecasting can take into account factors that can have a profound effect, single-factor models remain the most popular for determining the market capacity and consumption level. While using these models, the most significant factor one should be discriminated which takes into account the change in the market capacity. Thus, the implementation of correlation
regression analysis according to the series of the values of the market capacity and an independent macroeconomic parameter, that mean that the formation of the functional dependencies of the general form $S = f(x_i)$. $x_i$ is the $i$-th factor that affects the market capacity) enables identifying a statistically grounded relationship between the target variables.

Single-factor models of the predictive estimates of the market capacity should be applied in short-term time intervals during which only one target factors substantially changes and all other factors remain or are considered to remain unchanged or their change causes a slight change in the market capacity within the permissible statistical errors.

Single-factor models have their advantages while being applied:
- the differentiation of goods into main groups, selection and justification of the specific form of the functional dependence of the market capacity for each group of goods on the level of per capita income enables reflecting the peculiarities of consumers’ preferences of various types of goods and services and providing the most accurate modelling of consumer behavior in solving the tasks of the market capacity forecasting;
- for specific forecasts, such models enable determining which goods belong to the category associated with stocks and which ones are related to the category associated with habits.

But single-factor models also have a drawback – while they are being formed and justified, the vector of the impact on the dynamics of the market capacity of only one macroeconomic parameter can be taken into account on the basis of the predictive estimates of the market capacity. At the same time, other significant factors of market conditions are excluded from the analysis and neglecting these factors reduces the degree of the accuracy of the forecast estimates.

Single-factor models of the dependence of the demand (the market capacity) on prices or on income are widely used. Studies show that it should be kept in mind that the consumption of goods and services by a family over a certain period of time depends not only on its incomes within the target period but also on the previous incomes and on the incomes expected in the future. The amount of expenses also differs depending on the wish to save a part of the income or, conversely, to spend savings and take loans. That is, within a limited time interval, the statement “expenses are determined by the income level” is not correct; it is necessary to consider the long-term cash balance of a family taking into account planned purchases and current savings.

To study the change in demand depending on the income of various consumer groups, mainly two types of models are used – Engel functions and Törnqvist functions.

In the XIX century the German scientist Ernst Engel substantiated the economic law according to which the behaviour of consumers is related to the amount of their income and, as the incomes of the population’s consumption of goods increase, it grows disproportionately. Costs for food is growing more slowly than costs on durable goods, travel or savings and the structure of food consumption is changing toward better products. The growth of incomes leads to an increase in the share of savings and consumption of high-quality goods and services with a decrease in low-quality goods. The law is very outdated but Engel proved that empirical dependence is confirmed by a long experience of economic development.

Engel functions are known to be static models $S = aI^E$. Engel curves characterize the dependence of consumption (the market capacity) of various types of goods on income:
- for the items of primary necessity the index $E < 1$, that means that when the income increases, the extra costs for the goods of this category is a past that constantly decreasing (fig. 2, curve 1);
- for the durable goods the index of elasticity $E = 1$, that means that a share of costs for these goods in the extra income remains approximately unchanged (fig. 2, curve 2);
- for luxury goods the index of elasticity $E > 1$; that means that when the income increases greatly, its bigger share is spent for the goods of this group (Fig. 2, curve 3).

Fig. 2. Engel curves

According to Engel’s law, the elasticity of purchasing basic necessities from casual income is less than from the constant share of income. In particular, food costs are insensitive to income fluctuations. For luxury goods, the consumption of which is closely related to unforeseen incomes, the elasticity of purchasing from casual income is higher than from the constant income share.

But studies have shown that the tendencies of the Engel curve in developing countries differ from the trend of the curve of developed countries. When the level of total income is low, the overall elasticity of spending on food is elastic but if the total amount of costs increases, the total elasticity of food costs becomes inelastic. That is, the category of essential necessity is replaced by the category of luxury goods for certain market segments.

The Ukrainian society is a rather complex socially differentiated structure that was developed by N.M. Rymashevska, a specialist in the field of social demography and economic sociology and consists of ten social strata and at least three major social classes – “upper”, “middle” and “lower” ones [14]. Many sociological surveys testify that about half of Ukrainians...
are considered to be middle class but experts believe that only 10% of the population can be classified as middle class in Ukraine according to two basic indicators – the standard of living and education [15, 16], while 75–80% of the population can be referred to the lower class. In developed countries, the middle class includes the population that receives average incomes typical of a target society and this is the majority of the society (60–80%), so they can influence the overall direction of the development of a country and on the level and structure of consumption of goods and services.

The pattern of the dependence of demand on income was mathematically described in the studies of L. Törnqvist [17], these are special Törnqvist curves that are constructed according to Engel’s law. According to this approach, the whole set of goods and services can be submitted by three main groups; in each group, the relationship between the market capacity and consumer income is expressed by a certain form of dependence.

1. The goods of primary necessity, for which the curve that reflects the change of the market capacity (the change of demand for the necessities $S$ when the income $I$ increases) asymptotically approaches the upper boundary, which characterizes the level of saturation $S_{max}$ with these goods (fig. 3, curve 1).

![Engel and Törnqvist curves for various groups of goods](image)

**Fig. 3.** Engel and Törnqvist curves for various groups of goods:
1 – the goods of prime necessity; 2 – the goods of secondary necessity; 3 – luxury goods

The coefficient of elasticity of consumption of necessities varies from zero to one. The dependence of the market capacity on the income level of this group of goods is as follows

$$S_1 = \frac{S_{max} I}{I + a_1}, \quad (10)$$

where $S_1$ is the market capacity of the target group of goods or services of primary necessity; $S_{max}$ is the upper boundary of the consumption of necessities (maximum market capacity, the market potential of a target good of primary necessity): $\lim_{I \to \infty} S_1 = S_{max}^1$; $a_1$ is a model parameter.

2. Goods of the secondary necessity, for which the curve of the market capacity is convex (fig. 3, curve 2) and, when incomes increase, it approaches the upper boundary of consuming a certain group of goods $S_{max}$ that has greater value for these goods than for the goods of primary necessity; the demand for this group of goods occurs after the income reaches a certain amount $I \geq I_2$ and then the goods of this group can be purchased. The elasticity of demand for such goods approaches 1. The dependence of the market capacity $S_2$ on per capita income for the goods of the secondary necessity is as follows

$$S_2 = \frac{S_{max}^2 (I - I_2)}{I + a_2}, \quad (11)$$

where $S_{max}^2$ is the upper boundary of consuming the goods of the secondary necessity: $\lim_{I \to \infty} S_2 = S_{max}^2$; $I_2$ is the boundary value of income (if $I < I_2$, the market capacity of the goods of the secondary necessity $S_2 = 0$); $a_2$ is the parameter of a model.

3. Luxury goods, the consumption of which does not have the upper boundary and increases more rapidly when the income increases, it occurs only after the income exceeds the lower boundary value $I \geq I_3$, goods are not purchased (there is no demand) until this value is achieved. The curve of the market capacity of luxury goods $S_3$ is concave (fig. 3, curve 3) and is described by the following functional

$$S_3 = \frac{S_{max}^3 (I - I_3)}{I + a_3}, \quad (11)$$

where $I_3$ is the boundary value of income (is $I < I_3$, the market capacity of luxury goods $S_3 = 0$); $a_3$ is the parameter of a model.

When the values of income $I$ are great enough
That means almost all income growth can be spent on luxury goods in this situation.

Another curve $S_i = a_i I - b_i I^2$ for goods or groups of goods that are not used when a certain level of income is reached can be added to the Törnqvist functions. So, the demand for such products increases at first, reaches the maximum for $I = a_i / 2b_i$ and then starts decreasing up to 0, that means that there is no demand for such products when incomes are greater than this boundary.

In their works, A. Maki and M.A. Kamwe [18] on the basis of theoretical and empirical data considered and proved the hypothesis alternative to the Törnqvist-Wold hypothesis on the definition of luxury goods and necessities using the elasticity of the demand for general costs, direct and cross-price elasticity in the context of recent empirical observations taking into account the stratification of society in different countries.

Let us consider the available approaches to using the Törnqvist functions. Thus, in their works, H. Wold and L. Juréen [19] classify goods as necessities when the coefficient of direct price elasticity is negative, necessities and luxury goods are classified using the combination of prices and general income elasticity according to Törnqvist approaches, as “Törnqvist groups the commodities into “necessities” and “luxuries”, with price elasticity below and above unity, respectively. This leads to the conclusion that, as a rule, income elasticities of necessities are smaller than their price elasticities, whereas income elasticities of luxuries are greater than their price elasticities” [19, p.115].

This phenomenon can be explained by the fact that in the elasticity of demand there are two important characteristics of the complete demand system (and the demand functions of homogeneous products are determined by zero degree):

1) the sum of total cost elasticity, direct price elasticity and cross-price elasticity (cross-elasticity) for the i-th element is equal to zero

$$\frac{E_{Q_i}}{E_X} + \frac{E_{Q_i}}{E_{P_i}} + \ldots + \frac{E_{Q_i}}{E_{P_n}} + \ldots + \frac{E_{Q_i}}{E_{P_n}} = 0,$$  \hspace{1cm} (13)

where $E_{Q_i} = \frac{\partial \log Q_i}{\partial \log P_i}$, $j = 1, \ldots, n$ and $E_{Q_i} = \frac{\partial \log Q_i}{\partial \log X}$.

2) if the weight factor is taken into account, the mean value of the demand elasticity with account for the price of a fixed item of goods is equal to a negative proportion between the costs for the i-th item of goods and general costs.

$$\sum_j \left( P_j Q_j \right) \frac{E_{Q_i}}{E_{P_i}} = - \frac{P_j Q_j}{X}.$$  \hspace{1cm} (14)

For two items of goods, for example, food and non-food items, according to the formula (13), the following system of equations is true

$$\begin{align*}
\frac{E_{Q_1}}{E_X} + \frac{E_{Q_2}}{E_{P_2}} + \ldots + \frac{E_{Q_n}}{E_{P_n}} &= 0 \\
\frac{E_{Q_2}}{E_X} + \frac{E_{Q_2}}{E_{P_2}} + \ldots + \frac{E_{Q_n}}{E_{P_n}} &= 0.
\end{align*}$$  \hspace{1cm} (15)

According to the direct price elasticity of the first item of goods (food), the demand is inelastic; thus this product can be classified as a necessity according to the Törnqvist-Wold hypothesis. If the characteristics of the first product are focused on, according to (15), the Törnqvist-Wold hypothesis is obtained as the following system of equations

$$\begin{align*}
\left( P_1 Q_1 \right) \left( \frac{E_{Q_1}}{E_{P_1}} \right) + \left( P_2 Q_2 \right) \left( \frac{E_{Q_2}}{E_{P_2}} \right) &= -P_1 Q_1, \\
\left( P_1 Q_1 \right) \left( \frac{E_{Q_1}}{E_{P_1}} \right) + \left( P_2 Q_2 \right) \left( \frac{E_{Q_2}}{E_{P_2}} \right) &= -P_2 Q_2.
\end{align*}$$  \hspace{1cm} (16)

According to the second equation of the system (16), the following is true

$$\frac{E_{Q_1}}{E_{P_1}} = - \frac{P_2 Q_2}{P_1 Q_1} \left( \frac{E_{Q_2}}{E_{P_2}} + 1 \right).$$  \hspace{1cm} (17)

Let us consider these two cases.

1) if the demand for the second item of goods (non-food group) is elastic by own price and cross-price,

$$\frac{E_{Q_2}}{E_{P_2}} > 0$$

and, hence,

$$\frac{E_{Q_2}}{E_{P_2}} > \frac{E_{Q_2}}{E_{P_2}},$$

if $\frac{E_{Q_2}}{E_{P_2}}$ is changed in the first equation of the formula (15). That leads to the conclusion of the Törnqvist –Wold hypothesis – as a rule, the income elasticity of demand for necessities is less than their price elasticity of prices.

2) if the demand for the second item of goods is inelastic in its own price but elastic in cross-price,

$$\frac{E_{Q_2}}{E_{P_2}} < 0$$

that leads to the following conclusion of the Törnqvist –Wold hypothesis – the income elasticity of demand for necessities is greater than their own (direct) price elasticity.

In case of the n-dimension product case

$$\frac{E_{Q_1}}{E_X} + \frac{E_{Q_2}}{E_{P_2}} = - \left( \frac{E_{Q_1}}{E_{P_1}} + \frac{E_{Q_2}}{E_{P_2}} + \ldots + \frac{E_{Q_n}}{E_{P_n}} \right).$$  \hspace{1cm} (18)

When the sum of general elasticity and costs and own price elasticity is negative $\frac{E_{Q_1}}{E_X} < -\frac{E_{Q_1}}{E_{P_1}}$, the following inequality occurs...
This indicates that the sum of the cross-price elasticities is negative and the i-th product is classified as a luxury one. But to obtain the mathematically exact relationship between the cross-price elasticity $E_{ij}$, $(i \neq j, j = 1, 2, ..., n)$ and direct (own) price elasticity $E_{ii}$, $(j = 1, 2, ..., n)$ for all the goods available at the market is rather difficult. There are also difficulties in estimating the elasticity of the cross-price for the complete product assortment. For these reasons, the Thörnqvist-Wold hypothesis can be studied and substantiated only empirically using actual observations and accumulated statistics.

It should be also noted that other empirical laws of consumption were found on the basis of Engel’s law [20]:

Schwabe’s law – the poorer the family, the more the share of income is spent on housing;

Wright’s law – the higher the level of savings and the share of savings in costs;

Ginny’s law – if food costs grow or decrease in the arithmetic progression, the other types of costs change in the opposite direction but in the geometric progression.

Conclusions

While forecasting the market capacity, the following things should be taken into account – current purchases of durable goods depend not only on the income and prices but also on the stock of goods from consumers and substantially on a number of consumers and structural shifts in time; when consuming short-term goods, the product habit is of decisive importance; it is important to know which goods belong to the first category (related to stocks) and which ones belong to the second (related to habits), what goods of a certain segment are of primary necessity while they are considered as luxury ones in another group.

When developing and applying models, the following limitations should be taken into account – in some cases, the actual values of the market capacity at the previous moments of time that are used to build the trend model cannot be approximated by any one of the traditional functional dependencies (linear, static, logarithmic, exponential, ) due to the following factors – there are significant fluctuations in the change in the rate of growth in the development of the market; a spontaneous decrease in the market capacity over time; crisis phenomena; the group of products develops under the strong influence of various factors at different times, which leads to the stochastic consumption of goods.

Taking into account the mentioned above while developing of models for forecasting the market capacity enables describing the behaviour of consumers when purchasing the main groups of goods and services mathematically and estimating the consequences of the change in various factors, taking into account their influence on the market capacity of specific groups of goods and services in the most accurate way; determining the critical values of factors due to which the consumption of a certain group of products can be absolutely stopped or reduced to a minimum level.

References


ВУЗИ

Аналіз впливу на розвиток і величину місткості ринку математичних моделей прогнозування місткості ринку; розглянути необхідність урахування в прогнозних моделях структурних зрушень в чисельності споживачів за віковим проміжком.

Мета: методика визначення місткості ринку в загальних умовах внаслідок певних змін на ринку. Постати відомості про авторів / Сведения об авторах / About the Authors

Голованова Маїя Анатоліївна – кандидат технічних наук, доцент, Національний аерокосмічний університет ім. М. Є. Жуковського, доцент кафедри економіки та маркетингу, м. Харків, Україна; e-mail: m.golovanova@khai.edu; ORCID: 0000-0002-5534-6186.

Голованова Маїя Анатоліївна – кандидат технічних наук, доцент, Національний аерокосмічний університет ім. Н. Е. Жуковського, доцент кафедри економіки та маркетингу, тел. Харків, Україна; e-mail: m.golovanova@khai.edu; ORCID: 0000-0002-5534-6186.

Голованова Майя Анатольевна – PhD (Engineering), Associate professor, National Aerospace University "Kharkiv Aviation Institute", Associate Professor at the Department of Economics and Marketing, Kharkiv; e-mail: m.golovanova@khai.edu; ORCID: 0000-0002-5534-6186.

Лебедченко Віра Віталіївна – Національний аерокосмічний університет імені М.Є. Жуковського "ХАІ", старший викладач кафедри економіки та маркетингу, м. Харків, Україна; e-mail: veralebdencheko@gmail.com; ORCID: 0000-0003-2180-9063.

Лебедченко Віра Витальевна – Национальный аэрокосмический университет имени Н.Е. Жуковского "ХАИ", старший преподаватель кафедры экономики и маркетинга, г. Харьков, Украина; e-mail: veralebdencheko@gmail.com; ORCID: 0000-0003-2180-9063.

Lebedchenko Vira – National Aerospace University – Kharkiv Aviation Institute, Senior Lecturer of the Department of Economics and Marketing, Kharkiv, Ukraine; e-mail: veralebdencheko@gmail.com; ORCID: 0000-0003-2180-9063.

ВИКОРИСТАННЯ ЕКОНОМІКО-МАТЕМАТИЧНИХ МОДЕЛІЙ ДЛЯ ВИЗНАЧЕННЯ МІСТКОСТІ РИНКУ

Предмет: економіко-математичні моделі прогнозування місткості ринку з урахуванням системи впливаючих факторів. Мета: систематизація та обґрунтування теоретичних і методологічних підходів до прогнозування місткості ринку в умовах сучасного нестабільного стану економіки. Завдання: дати найблизьші повні систематизований перелік факторів, що впливають на розвиток і величину місткості ринку; розглянути особливості, переваги та недоліки застосування існуючих економіко-математичних моделей прогнозування місткості ринку; розглянути необхідність урахування в прогнозних моделях структурних зрушень в чисельності споживачів за віковими проміжками; виявити особливості впливу споживачів товарів розкішних та товарів першого необхідності на змінювання величини місткості ринку. Методи: системний аналіз, факторний аналіз, регресійно-кореляційний аналіз, методи кумулятивних кривих. Результати: розглянути принципи побудови економіко-математичних моделей прогнозування місткості ринку. Систематизовано фактори, які впливають на
місткість ринку на мікро- та макроуровнях та мають випадковий або систематичний характер впливу. Виявлено переваги, недоліки та особливості застосування багатофакторних та однофакторних моделей місткості ринку, зокрема кумулятивних кривих. Запропоновано обов’язкове урахування у багатофакторних моделях демографічних чинників (змінення структури вікових когорт у часі). Показано альтернативне використання кумулятивних кривих попиту щодо груп товарів розкіші та першої потреби в залежності від доходу. Висновки. Урахування структурних зрушень кількості споживачів у часі, особливостей споживання товарів розкіші та товарів першої необхідності із застосуванням коефіцієнтів еластичності попиту за доходом та інших значущих факторів, дасть змогу математично описати споживання основних груп товарів і послуг, спрогнозувати величину місткості їх ринку; визначити критичні значення факторів, за яких можливим є повне припинення споживання певної групи продукції або зменшення споживання до мінімального рівня.

Ключові слова: ємність ринку, економіко-математична модель, еластичність попиту, доход, споживання, ціна, попит.

### ІСПОЛЬЗОВАНИЕ ЭКОНОМИКО-МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ДЛЯ ОПРЕДЕЛЕНИЯ ЁМКОСТИ РЫНКА

**Предмет:** экономико-математические модели прогнозирования ёмкости рынка на основе системы влияющих факторов.

**Цель:** систематизация и обоснование теоретических и методологических подходов к прогнозированию ёмкости рынка в условиях современного нестабильного состояния экономик.

**Задачи:** дать наиболее полный систематизированный перечень факторов, влияющих на развитие и величину ёмкости рынка; рассмотреть особенности, преимущества и недостатки применения существующих экономико-математических моделей прогнозирования ёмкости рынка; рассмотреть необходимость учёта в прогнозных моделях структурных сдвигов в численности потребителей по возрастным возрастам; выявить особенности влияния потребления товаров роскоши и товаров первой необходимости с учётом дохода на изменение ёмкости рынка.

**Методы:** системный анализ, факторный анализ, регрессионно-корреляционный анализ, методы кумулятивных кривых.

**Результаты.** Рассмотрены принципы построения экономико-математических моделей прогнозирования ёмкости рынка. Систематизированы факторы, влияющие на ёмкость рынка на микро- и макроуровне и имеющие случайный или систематический характер воздействия. Выявлены преимущества, недостатки и особенности применения многофакторных и однофакторной модели ёмкости рынка, в частности кумулятивных кривых. Предложен обязательный учёт в многофакторных моделях демографических факторов (изменение структуры возрастных когорт во времени). Показано альтернативное использование кумулятивных кривых спроса по группам товаров роскоши и первой необходимости в зависимости от дохода.

**Выводы.** Учёт структурных сдвигов количества потребителей во времени, особенностей потребления товаров роскоши и товаров первой необходимости с применением коэффициентов эластичности спроса по доходу и других значимых факторов, позволит математически описать потребления основных групп товаров и услуг, спрогнозировать величину ёмкости рынка; определить критические значения факторов, при которых возможно полное прекращение потребления определённой группы продукции или уменьшение потребления до минимального уровня.

**Ключевые слова:** ёмкость рынка, экономико-математическая модель, эластичность спроса, доход, потребление, цена, спрос.